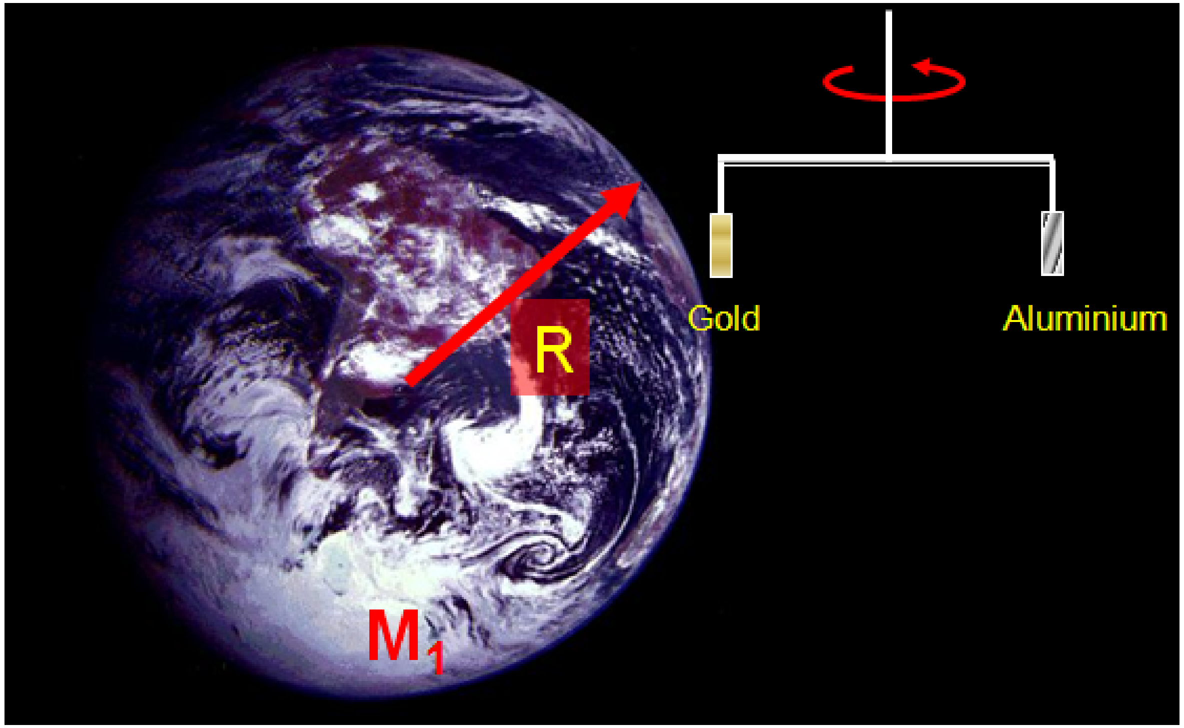
The Equivalence Principle: A Question of Mass



Summary

In this Activity, we will investigate:

- gravitational acceleration;
- Galileo's experiments with pendulums and inclined planes;
- the differences between inertial mass and gravitational mass; and
- the Weak Equivalence Principle.

The story so far...

Let's recap a couple of important results that we will need for this Activity.

- **Galileo** showed that objects with different masses and compositions fell at *exactly the same rate* under gravity.
- **Newton's law of universal gravitation** states that gravity is a property *created* and *experienced* by all objects which have *mass*.
- **Newton's second law of motion** depends on the *mass* of the object which is being *"forced"* to accelerate.

So *mass*, whatever it is, seems to be a very important part of gravitation.

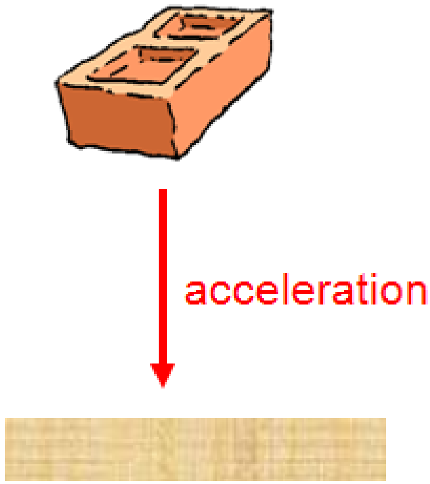
Gravitational acceleration

If an object is dropped in a gravitational field, its velocity will increase: the gravitational force causes an acceleration.

Near the surface of the Earth, the acceleration due to gravity is very close to 9.8 metres per second per second (directed towards the centre of the Earth).

This is often written as:

$$g = 9.8\ \mathrm{ms}^{-2}$$

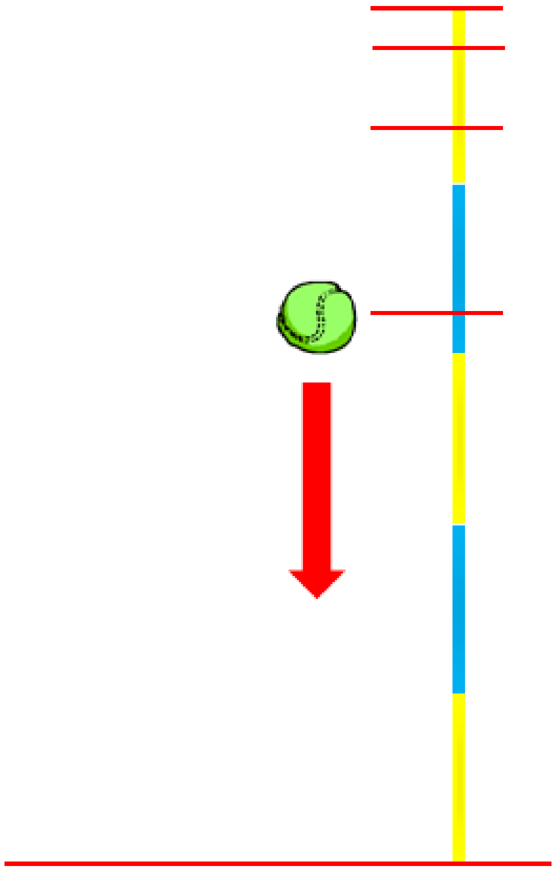


After letting go, an object will have a speed of:

9.8 m s^{-1} after one second
19.6 m s^{-1} after two seconds
29.4 m s^{-1} after three seconds
39.2 m s^{-1} after four seconds
49.0 m s^{-1} after five seconds...

...unless it hits the ground first!

As the velocity increases, the vertical distance traveled each second also increases.



Galileo's experiment?

Could Galileo really have performed an experiment to show that objects fell at the same rate by dropping them from the Leaning Tower of Pisa? The Tower is around 56 metres high, so that a ball dropped from this height would take 3.4 seconds to reach the ground with a final speed of nearly 120 km hour^{-1}!

It would have been quite an effort to determine which object hit the ground first at this speed, especially without the benefit of photography or a stopwatch. Plus there was the added disadvantage of air resistance...

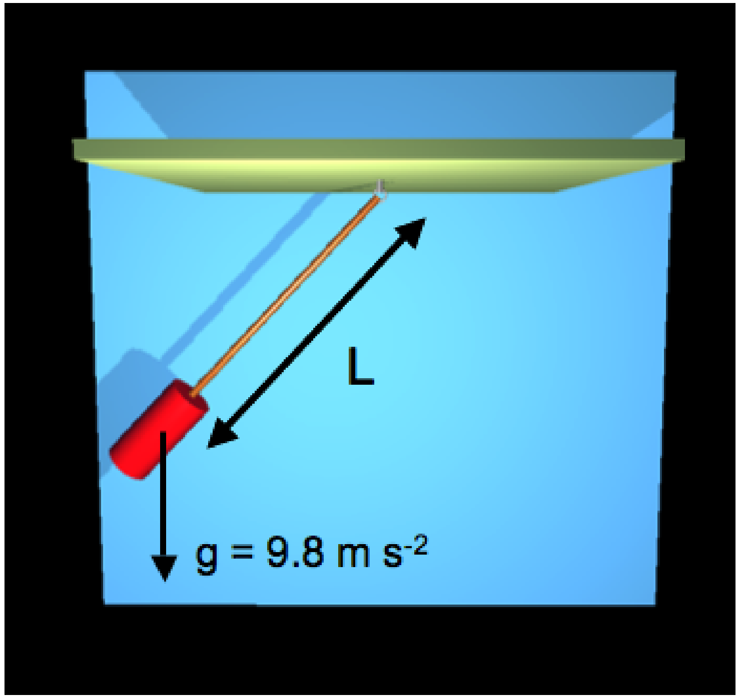
So if Galileo didn't perform his experiment at Pisa, how did he show that the gravitational acceleration was independent of mass?

Swinging into action

He used a pendulum!

The time it takes a pendulum to complete one swing (the period) depends on the length of the pendulum, L, and the acceleration due to gravity, g.

$$\text{Period} \propto \sqrt{\frac{L}{g}}$$



The more times the pendulum swings (total elapsed time, T_{total}), the more accurately you can measure the period,

$$1 \text{ swing gives 1 measurement, so } T = T_{total}$$
$$10 \text{ swings gives 10 measurements, so } T = T_{total}/10$$
$$100 \text{ swings gives 100 measurements, so } T = T_{total}/100$$

...in theory, anyway.

If it wasn't for air resistance and friction at the join between the pendulum and the ceiling, the pendulum would keep swinging forever with the same period. In reality, the period slowly changes as the height of each upswing decreases.

Galileo was able to show that the gravitational acceleration was the same for all objects by replacing the "bob" on the pendulum with different masses. Regardless of the mass or composition of the "bob", the period remained the same!

9.8 metres per second per second

Where does this mysterious value of 9.8 ms^{-2} come from? And how do we actually measure it?

Let's go back to Newton's universal law of gravitation:

$$F = \frac{GM_1M_2}{R^2}$$

We can write this as:

$$F = M_2 \times \left(\frac{GM_1}{R^2}\right)$$

This looks a lot like Newton's second law:

$$F = \text{mass} \times \text{acceleration}$$

Gravitational mass

If we define the gravitational acceleration produced by a body with mass M_1 as:

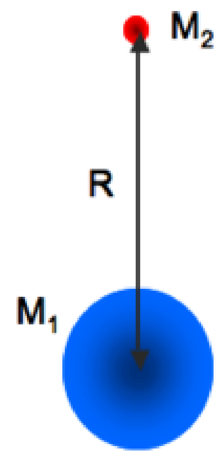
$$g = \frac{GM_1}{R^2}$$

then a body of mass M_2 will experience a force:

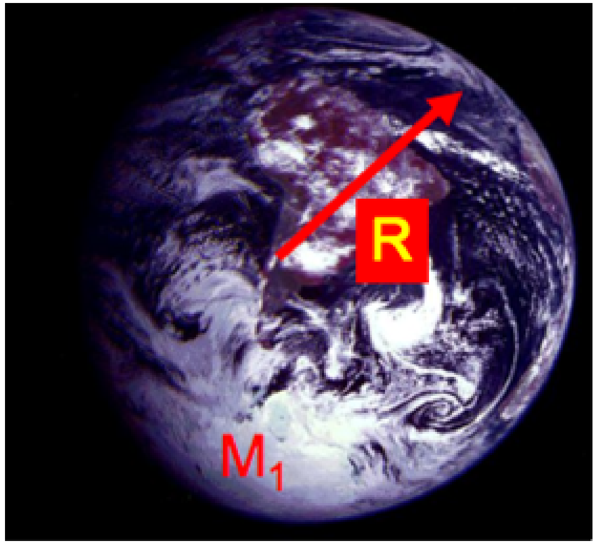
$$F = M_2 \times g$$

when it is a distance R from M_1.

The mass M_2 is called the gravitational mass of the body, as it determines how that object will respond to the gravitational field produced by the body with mass M_1.



If we take the values for the Earth:



$$\begin{aligned}\text{Mass} &= M_1 \\ &= 5.98 \times 10^{24}\ \text{kg} \\ \text{Radius} &= R \\ &= 6.38 \times 10^{6}\ \text{m} \\ G &= 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}\end{aligned}$$

and substitute them into:

$$g = \frac{GM_1}{R^2}$$

we get:

$$g = 9.8\ \text{ms}^{-2}$$

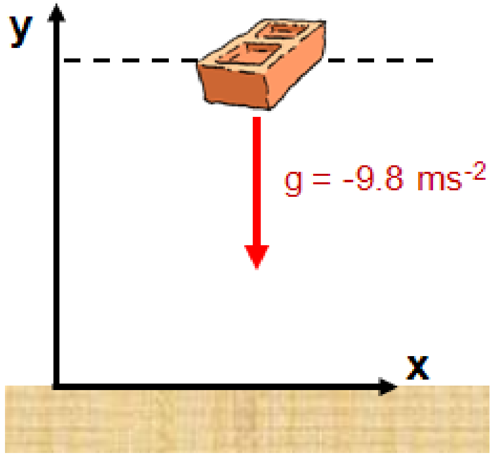
This is the gravitational acceleration at the surface of the Earth, and is directed towards the Earth's centre.

The value of g at the surface of the Earth changes a little depending on whether you are at the poles, the equator or somewhere in between as the Earth is not a perfect sphere.

Why the minus sign?

Just to sidetrack briefly, sometimes you will see g written with a minus sign as:

$$g = -9.8 \text{ ms}^{-2}$$



Suppose we draw a set of coordinate axes with the ground level at y = 0 (for convenience), and the positive y-axis pointing away from the ground.

If we have an object at some height above the ground, the acceleration due to the Earth's gravitational field causes the object to move towards more **negative values** of the y coordinate (i.e. it moves towards y = 0). We have to include the minus sign in order to get the direction of the acceleration correct, as acceleration is a **vector**.

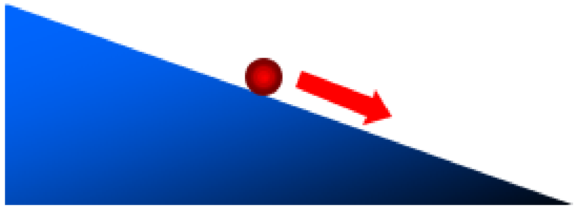
Mathematical notation

However... sometimes it can be a nuisance to incorporate the minus sign, as the choice of a y-axis pointing away from the ground is not always the best one to use. For example, if we had set $y = 0$ to be the location of the object we are about to drop, and had the y-axis pointing towards the ground, we do not need the minus sign in g!

For convenience (and to save us having to include a whole lot of extra mathematical notation!), we will use the symbol $g = 9.8\ \mathrm{ms}^{-2}$ to mean the magnitude of the gravitational acceleration. If we want to be explicit about the direction (which starts to matter when you are doing actual calculations), we will include the minus sign or draw an arrow to indicate the direction.

Experimenting with inclined planes

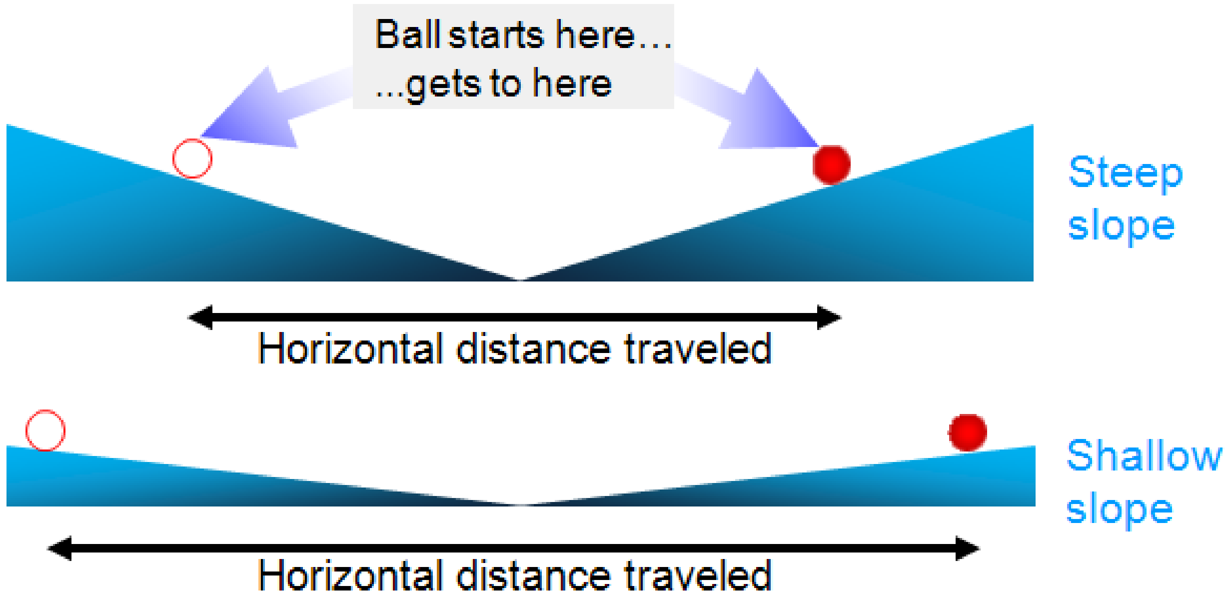
Another experiment which Galileo performed involved rolling balls down inclined planes.



The gravitational force causes the ball to accelerate down the slope, but at speeds which are much slower, and therefore easier to observe than objects in free-fall.

Galileo observed that by using two slopes, one for the ball to roll down and one for the ball to roll up, that the ball would reach almost the same height that it had started at.

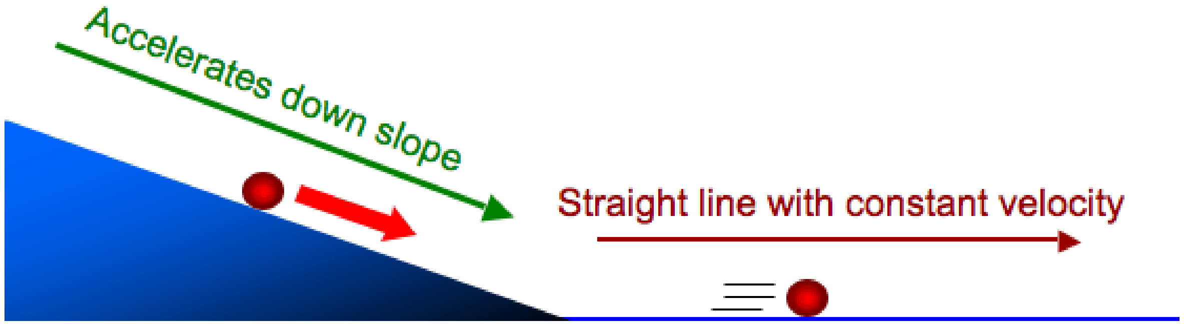
It doesn't quite get there because of friction, which takes away some of the ball's energy.



The shallower the slope, the greater the horizontal motion (if both balls start moving from the same height).

Inertia

Galileo reasoned that if there was no friction, when the ball reached the bottom of the slope it would continue to travel without requiring a force to propel it further.



The ball has a special property called **inertia** which causes it to keep moving in the absence of any force. This is in contrast to Aristotle's belief that objects did not move unless there was a force causing the motion.

Galileo's experiments with inclined planes provided the basis for Newton's First Law of Motion, also called the **Law of Inertia**.

Inertial mass

The **inertial mass** of an object is its resistance to a force. Any time a force is applied to a body in order to *change* its motion (that is, produce an acceleration), the body opposes that change through its inertial mass.

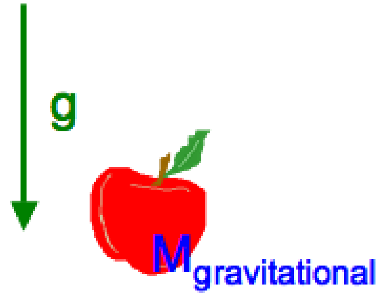
We should really write Newton's Second Law as:

$$F = \text{inertial mass} \times \text{acceleration}$$

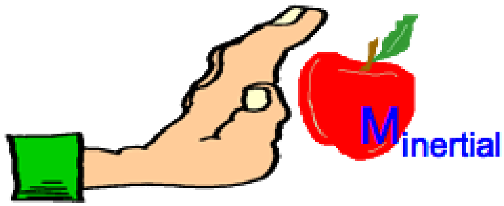
The greater the inertial mass, the harder it is to accelerate for a given strength of force.

Gravitational or inertial mass?

So, if I put an object in a gravitational field, it responds with its **gravitational mass**.



If I push an object, it responds to that push with its **inertial mass**.



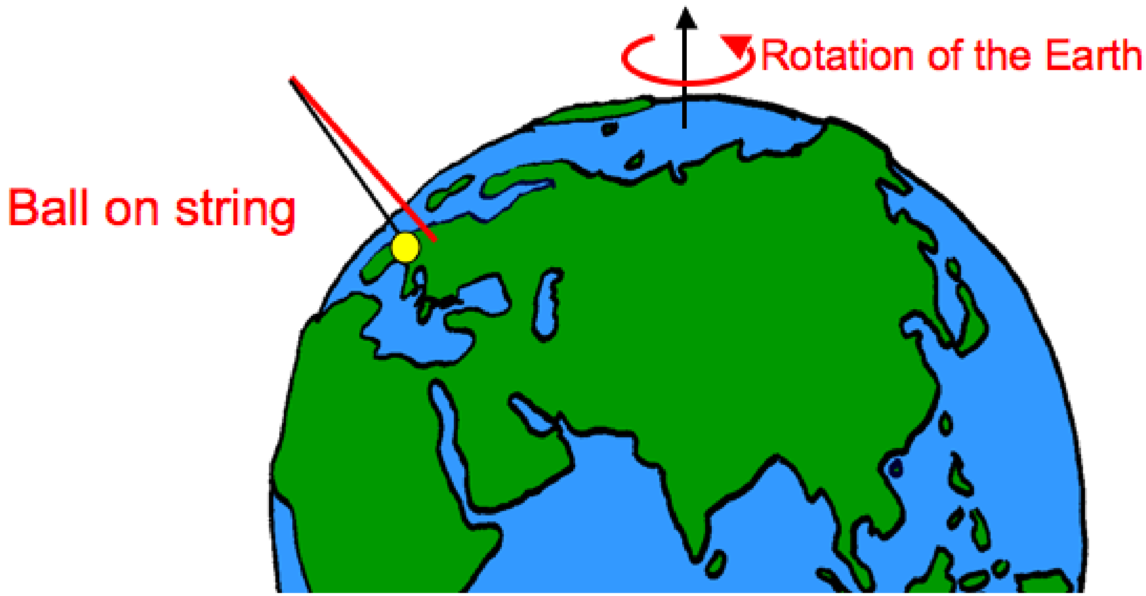
Discuss! Are these two masses the same? What are the arguments for and against this idea?

There is absolutely no reason why they have to be the same, but according to the best experiments physicists have been able to perform, they are.

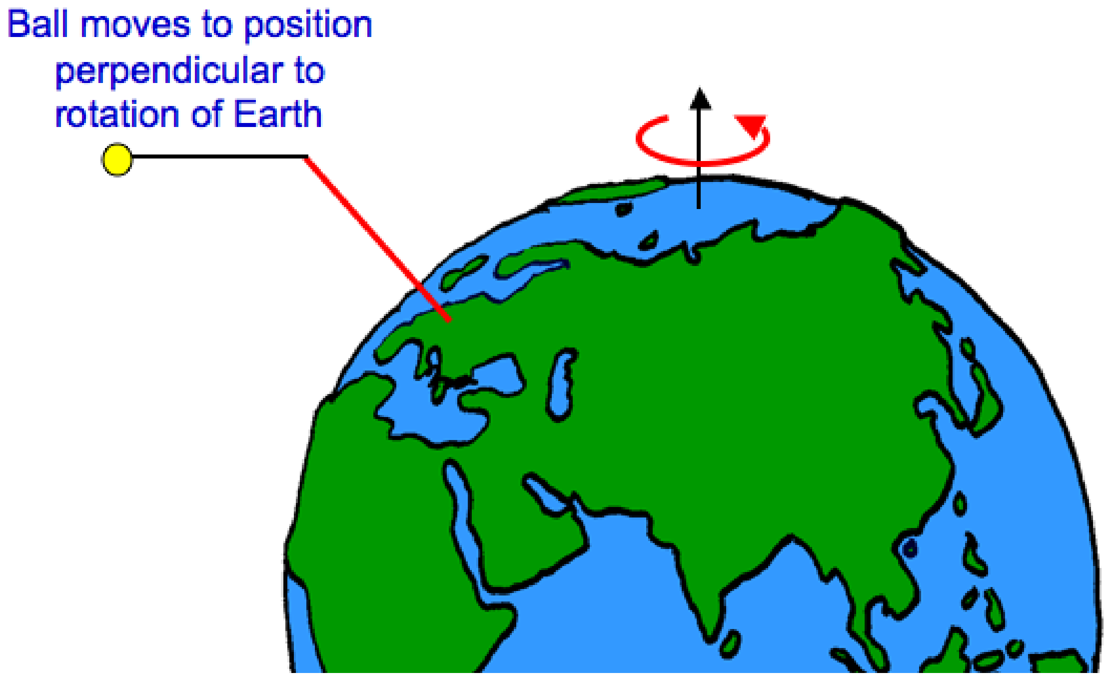
Lóránd Baron von Eötvös

At the turn of the 20th century, the Hungarian physicist Baron von Eötvös and his colleagues performed a series of experiments to determine whether the ratio of the inertial and gravitational masses of different substances were the same.

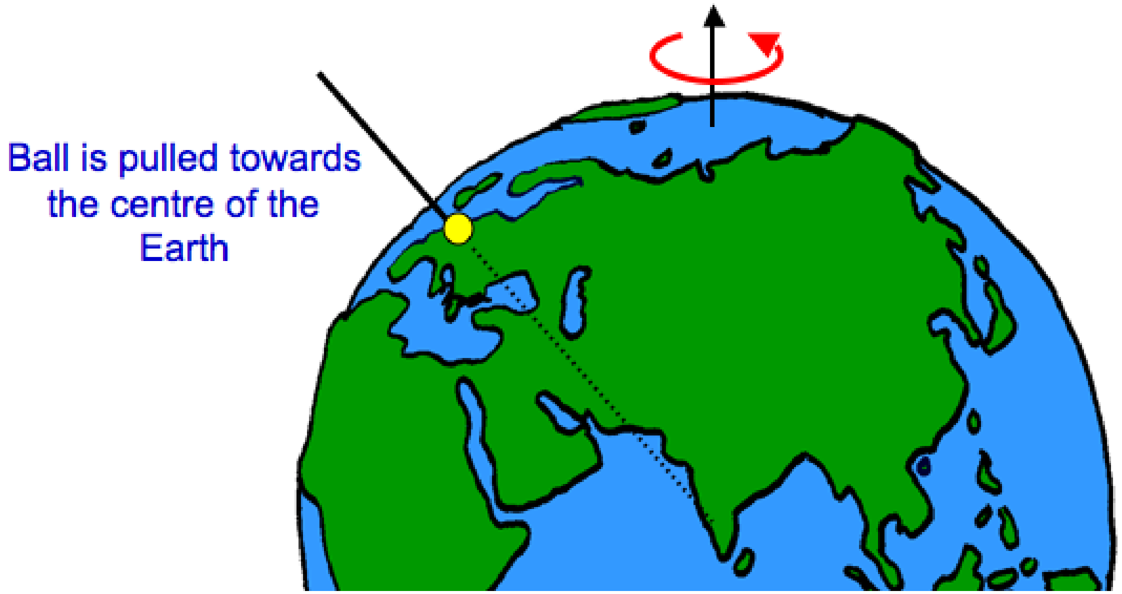
Consider a ball tied to a string with the other end attached to a pole in the ground.



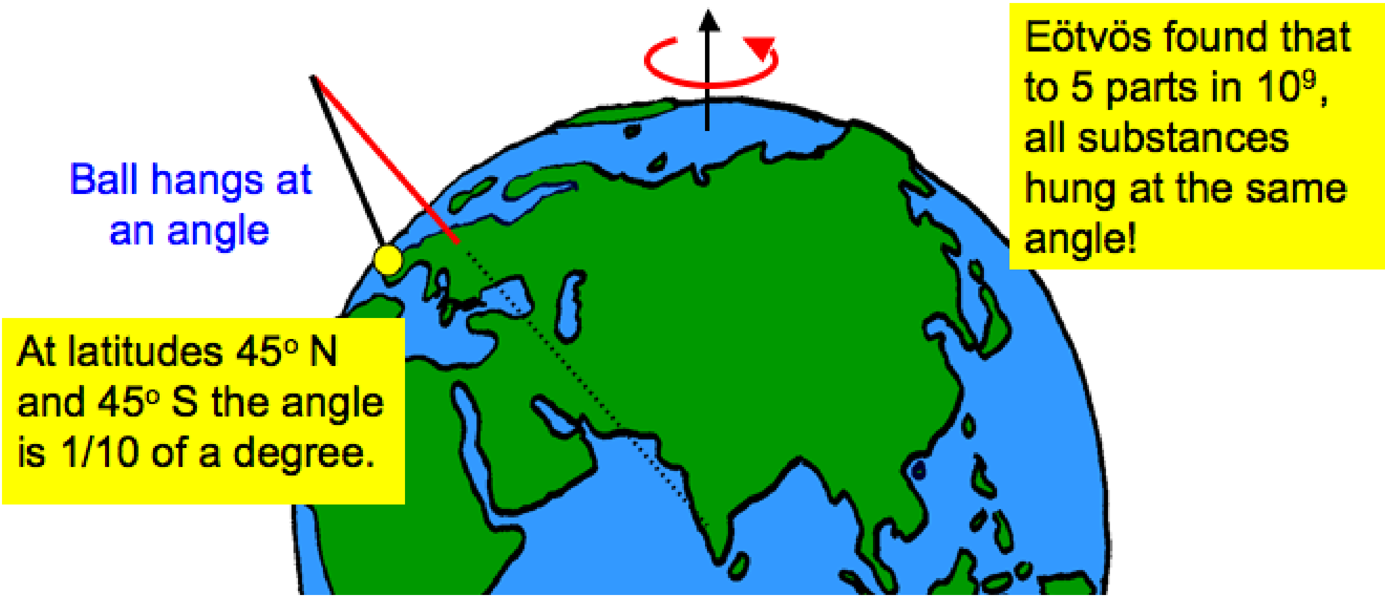
If the ball had no gravitational mass, its inertial mass would cause it to be flung out by the rotation of the Earth.



If the ball had no inertial mass, its gravitational mass would pull it towards the centre of the Earth.



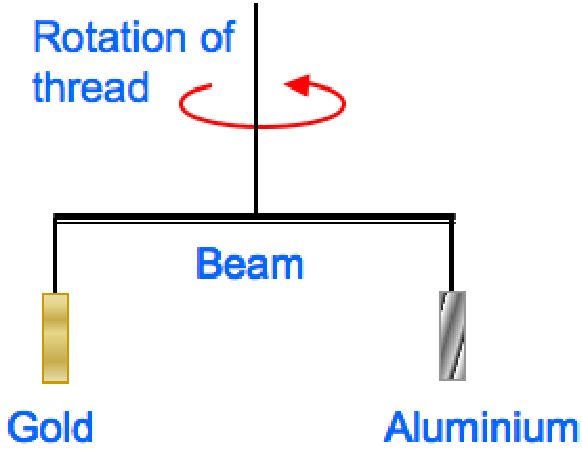
Since a real ball has both inertial and gravitational mass, it will hang at a slight angle.



Experimenting with torsion balances

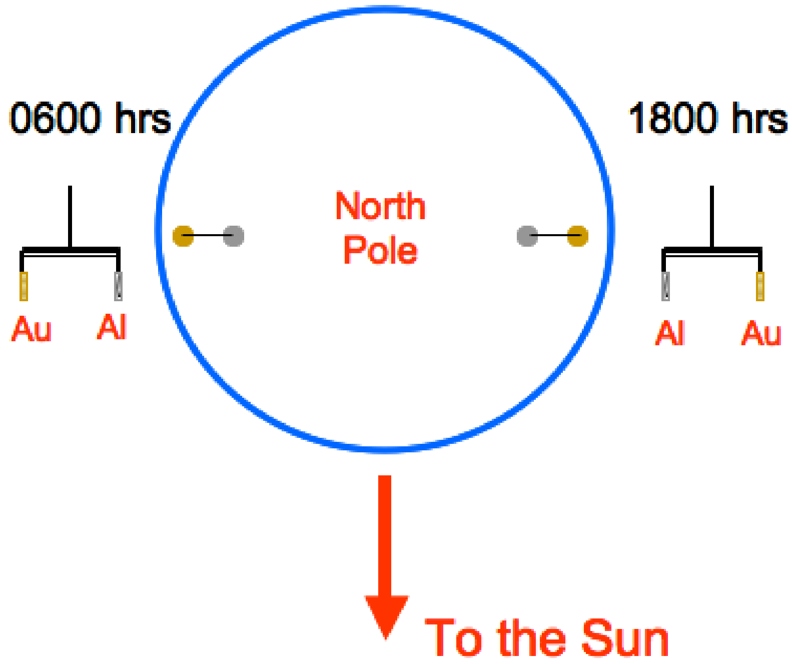
A similar idea to that of Baron von Eötvös was used by three physicists, P. G. Roll, R. Krotkov and R. H. Dicke, who performed an experiment in 1964 which compared the force of gravity due to the Sun on various materials.

In their experiment, they made use of a torsion balance.



A torsion balance allows a very accurate measurement of the rotation of a thread, to which two masses (in this case gold and aluminium) are attached via a beam.

Consider the view of the Earth looking down from the North Pole at the torsion balance. The experimental arrangement is shown at 0600 hours.



Twelve hours later, the Earth has rotated, and the position of the apparatus changes with it.

If there was a difference between the ratio $m_{gravitational}/m_{inertial}$ of the two materials, there would be a detectable rotation of the torsion balance over a 24 hour period.

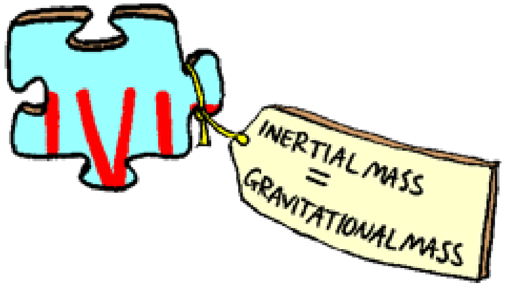
No detectable rotation was measured:

$$\frac{M_{\text{gravitational}}}{M_{\text{inertial}}} = 1$$

It's fundamental

Einstein looked on this equality of inertial and gravitational masses as something fundamental about the Universe. He called it the **Weak Equivalence Principle** (WEP) and it is one of the most important pieces of General Relativity.

The WEP says that: *the motion of a neutral test body released at a given point in space-time is independent of its composition.*

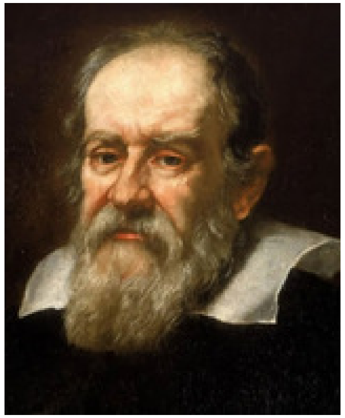


By neutral, we just mean that the body is only being acted on by gravity, and not by electrostatic or magnetic forces.

Experimental evidence for the WEP

Some of the most accurate experiments performed to test the Weak Equivalence Principle were performed by R. H. Dicke and V. B. Braginsky. They showed that gold, platinum and aluminium fall at exactly the same rate to one part in 10^{12}.

This is exceptional experimental evidence that the Weak Equivalence Principle is valid, and as Galileo suspected over 300 years earlier: *all materials fall at the same rate regardless of their composition.*



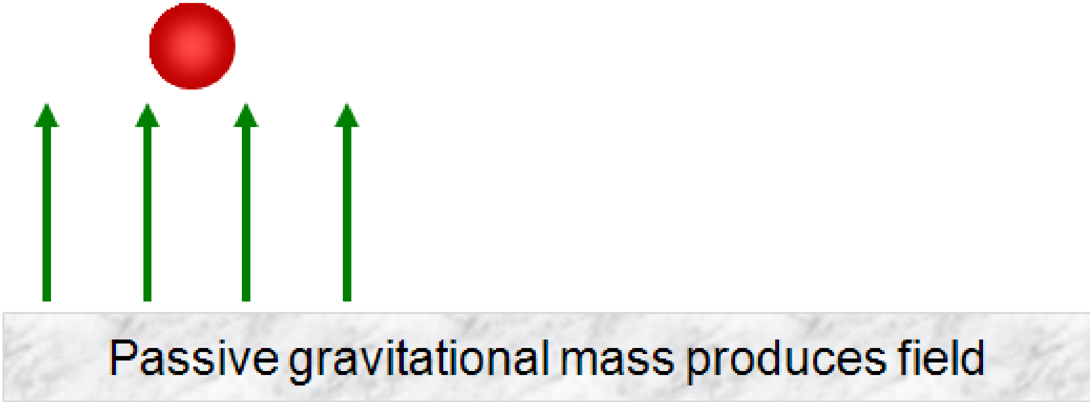
Active versus passive mass

We have now shown that there are two types of mass, gravitational mass and inertial mass, but these are identical according to the Weak Equivalence Principle.

We can go further and distinguish between two different types of gravitational mass:

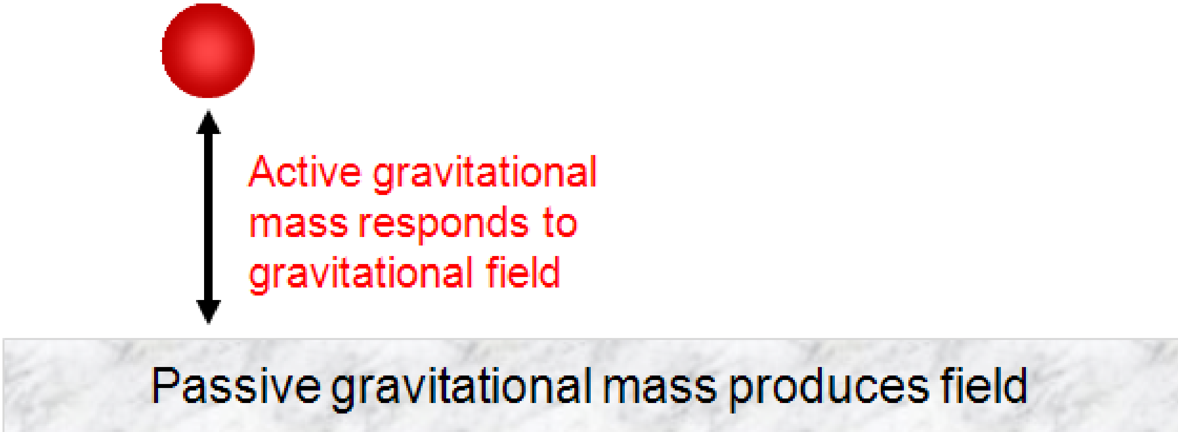
- **Active gravitational mass** is the response of an object to a gravitational field.
- **Passive gravitational mass** determines the strength of a gravitational field.

The gravitational field generated by the Earth is due to its passive gravitational mass.



The forces we experience as we walk around on the Earth's surface are a result of our active gravitational masses.

But, as with inertial and gravitational mass, the active and passive masses of a body are also identical.



More weighty matters

So, according to Einstein, **gravitational** and **inertial mass** are the same thing. This may seem obvious, but it is actually quite profound.

The Newtonian model of universal gravitation is not seriously affected if the gravitational and inertial masses of an object are different. But the **Weak Equivalence Principle** is fundamental to Einstein's development of his General Theory.

Before we leave mass to delve further into General Relativity, we need to consider one more thing: weight. Or more importantly for the next Activity, **weightlessness**.

Weight or mass?

How much do you weigh? It's a common enough question, and one which doesn't always get answered truthfully.

Suppose you were to stand on a pair of bathroom scales on the Earth, and find that your **apparent weight** is 70 kilograms. Now travel to the Moon with your bathroom scales. Miraculously enough, you now *weigh* about 1/6 of what you did on Earth. Celebrate by having some chocolate or ice-cream!

Credit: NASA

Next, take your scales with you to Jupiter, and stand on them again. Oh no! You now weigh nearly 3 times your Earth-weight. Maybe you shouldn't have celebrated quite so much on the Moon. . .

If you give your *apparent weight* in kilograms or pounds or stones, then you aren't quite telling the whole truth, because weight is a *force* and depends on the strength of the gravitational field acting on you.

Never leave your mass behind

Your bathroom scales measure how much of a downward force you are applying, and how much force the scales must push back at you (Newton's Third Law). According to Newton's Second Law, the force on you due to gravity is:

$$\text{Mass} \times \text{gravitational acceleration} = M \times g$$

Your scales were built on Earth, and so are calibrated against the gravitational acceleration at the surface of the Earth, $g = 9.8$ m s^{-2}.

Whether you are on the Earth, the Moon or Jupiter, your body contains the same amount of matter. Your apparent weight changes because the gravitational acceleration at the surface of the Moon and at Jupiter are not the same as for the Earth.

And strictly speaking, the correct units of weight (which is simply a force) is Newtons!

Credit: NASA, Johnson Space Center, 1972

Guaranteed weight loss

What if there was no gravitational acceleration, so that, locally, $g = 0$?

Then the force due to gravity is:

$$\text{Mass} \times g = 0$$

In other words, you would be **weightless**! But you would still have the same mass as before. In the next Activity, we will look at some things you can do to lose all of your weight, even when g is not zero...

Summary

In this activity we looked at early experiments of gravity, two different kinds of experiential masses (inertial and gravitational) and the formulation of the Weak Equivalence Principal.

In the next activity we're going to investigate gravity and acceleration in quite a bit more detail. We'll start by jumping in an elevator and going for a ride...