Quantum simulations of the early universe

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Outline

Novel quantum optics - engineered quantum systems

ULTRALOW temperatures to below $1nK$

TESTS QUANTUM THEORY IN NEW REGIMES!

- Quantum optics and integrated photonics
- Bose-Einstein condensates: atom ‘photons’.
- Quantum superfluid fermions: atom ‘electrons’
- Nano-mechanical oscillators
- Ion traps
- Superconducting quantum circuits

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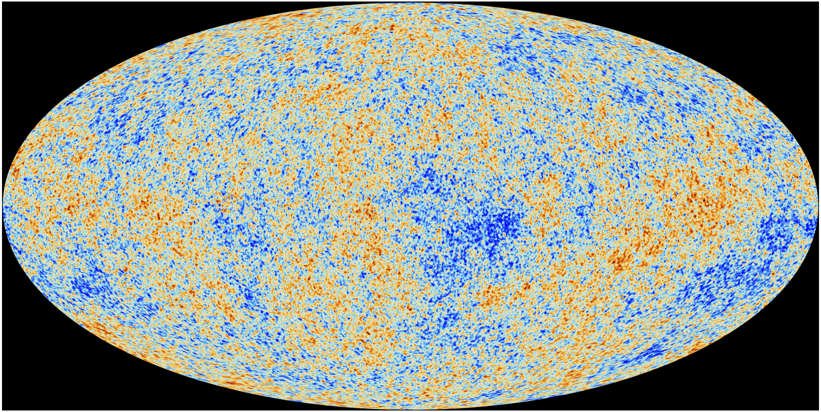
Theoretical challenges

Many interesting theoretical challenges - for example

- Simulations of quantum paradoxes
- Experimental analogs of the early universe

Early universe: dominated by quantum fluctuations? Planck CMB survey



Early universe quantum models

Early universe models

- The simplest model has a scalar inflaton field
- Relativistic, interacting quantum field dynamics
- $\phi(x)$ is described by the Lagrangian

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi),$$

where $V(\phi)$ is the potential down which the scalar field rolls

The original theory: Coleman's 'false vacuum'

PHYSICAL REVIEW D VOLUME 15, NUMBER 10 15 MAY 1977

Fate of the false vacuum: Semiclassical theory*

Sidney Coleman
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 24 January 1977)

It is possible for a classical field theory to have two homogeneous stable equilibrium states with different energy densities. In the quantum version of the theory, the state of higher energy density becomes unstable through barrier penetration; it is a false vacuum. This is the first of two papers developing the qualitative and quantitative semiclassical theory of the decay of such a false vacuum for theories of a single scalar field with nonderivative interactions. In the limit of vanishing energy density between the two ground states, it is possible to obtain explicit expressions for the relevant quantities to leading order in $\hbar$; in the more general case, the problem can be reduced to solving a single nonlinear ordinary differential equation.

Inflationary universe models

This is the start of more complex quantum models

- Need to assume a comoving, expanding frame
- Can include a background gravitational tensor
- Most inflation models have friction terms as well
- Also possible to have vector early universe fields
- First step: understand the simplest case

Problem:
quantum theory is exponentially complex!

BEC many-body states have exponentially large dimension

- Typical BEC: n particles distributed among m modes
  - take $n \simeq m \simeq 500,000$:
  - Number of quantum states: $N_s = 2^{2n} = 2^{1,000,000}$
- **More basis states than atoms in the universe**
- **Can't calculate with a $10^{300,000} \times 10^{300,000}$ matrix!**

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Proposed solution

Complementary study: quantum+phase-space simulation!

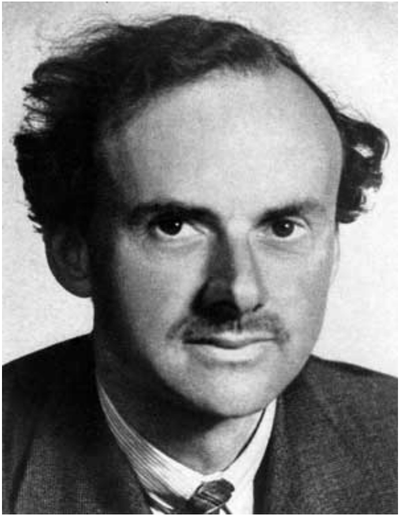
- Quantum emulation creates a laboratory model
- Phase-space simulation creates a computer model
- Each involves DIFFERENT types of approximation
- Can study BOTH and compare them
- Goes beyond the linearized approximation

Wigner-Moyal phase space methods

Simulates quantum observations - but not probabilistic!

- Generally doesn't give positive probabilities
- No stochastic evolution
- Can't always be sampled
- Not suitable for Bell violations
- Large occupation: 'truncated' Wigner

Dirac to Moyal: ‘Possible heavy criticism’



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Dear Moyal,

I heard from Bartlett that you would be willing to talk about your quantum theory work at our colloquium, and I think it would be a good idea to have it discussed if you do not mind possible heavy criticism. Would Friday the 25th Jan at 3 pm suit you? If this does not leave you sufficient time we could make it a week later. If you cannot conveniently deal with it all in one afternoon there is no objection to your carrying on the following week.

Yours sincerely,

P. A. M. Dirac.

Overcoming Dirac's objection

How do we overcome Dirac's objection?

- Take the case of large occupation numbers per mode
- Time-evolution equation can be truncated for $n \to \infty$
- Resulting Wigner distribution is approximately a probability

Sampled values like weak quantum measurements

- Advantage: Quantum simulations with existing computers!

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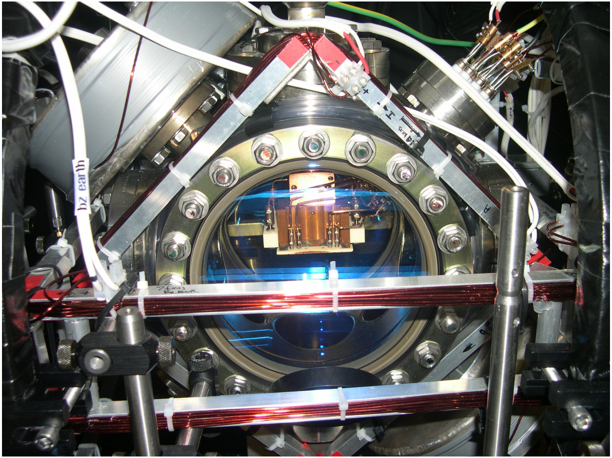
Test case: Interferometry on an atom chip (Siderov, Swinburne)



Coupled Bose gas model

A D-dimensional Bose gas with two spin components that are linearly coupled by an external microwave field. This system obeys the following nonrelativistic Heisenberg equation for $i = 1, 2$ and $j = 3 - i$:

$$i\hbar\partial_t\Psi_i = \left\{ -\frac{\hbar^2}{2m}\nabla_x^2 + g_i\Psi_i^\dagger\Psi_i + g_c\Psi_j^\dagger\Psi_j \right\}\Psi_i - \nu\Psi_j,$$

Here, ∇_x^2 represents the D-dimensional Laplacian operator, and g_i, g_c are the $D-$dimensional coupling constants. The field commutators are:

$$\left[\Psi_i(\mathbf{x}), \Psi_j^\dagger(\mathbf{x}')\right] = \delta_{ij}\delta^D(\mathbf{x} - \mathbf{x}')$$

Scaled equations

Result of operator mappings:

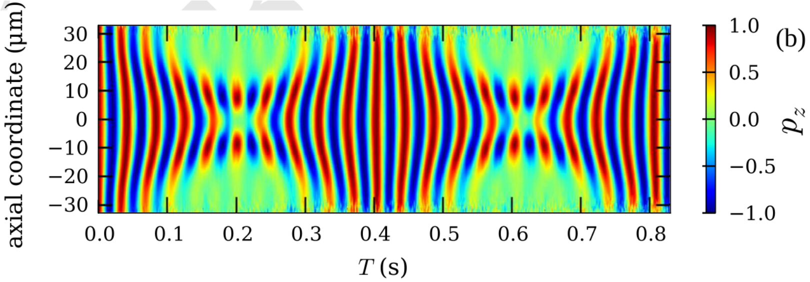
$$i\partial_\tau \psi_i = \left\{ -\frac{1}{2}\nabla^2 + \gamma \psi_i^\dagger \psi_i + \gamma_c \psi_j^\dagger \psi_j \right\} \psi_i - \tilde{\nu} \psi_j,$$

Scaling: $\tau = t/t_0$, $\zeta = x/x_0$,

$$t_0 = \hbar/gn$$
$$x_0 = \hbar/\sqrt{gnm}.$$

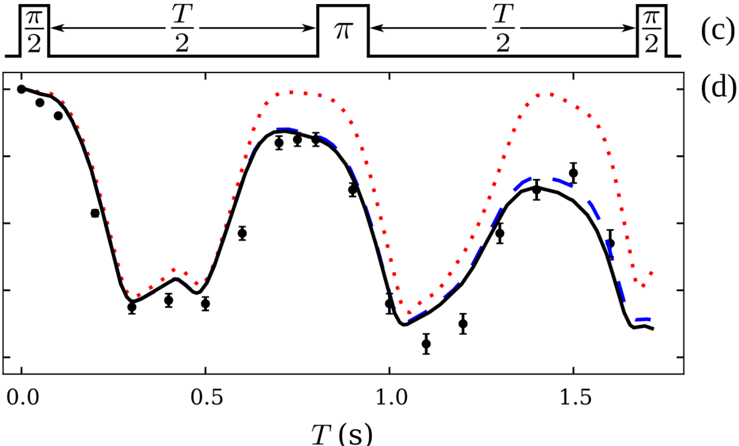
Interferometry experiments

A two-component, 4×10^4 atom ^{87}Rb BEC is in a harmonic trap with internal Zeeman states $|1, -1\rangle$ and $|2, 1\rangle$, which can be coupled via an RF field.



Truncated Wigner simulations replicate BEC fringe visibility

- Blue line = Wigner simulation, black line = Wigner + local oscillator noise, red dots = GPE, error bars are measured



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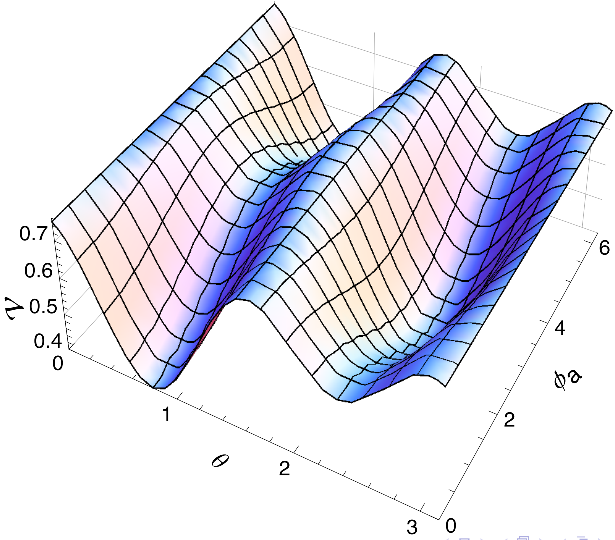
Stochastic time-evolution equations

$$i\partial_\tau \tilde{\psi}_1 = \left\{ -\frac{1}{2}\nabla_\zeta{}^2 + \gamma|\tilde{\psi}_1|^2 \right\} \tilde{\psi}_1 - \tilde{\nu}\tilde{\psi}_2,$$
$$i\partial_\tau \tilde{\psi}_2 = \left\{ -\frac{1}{2}\nabla_\zeta{}^2 + \gamma|\tilde{\psi}_2|^2 \right\} \tilde{\psi}_2 - \tilde{\nu}\tilde{\psi}_1. \qquad (1)$$

For the truncated Wigner calculations, with an initial coherent state of $\bar{\psi}(\zeta)$, so that $\tilde{\psi}(\zeta) = \bar{\psi}(\zeta) + \Delta\tilde{\psi}(\zeta)$, one would have:

$$\left\langle \Delta\tilde{\psi}(\zeta)\Delta\tilde{\psi}^*(\zeta') \right\rangle = \frac{1}{2}\delta\left(\zeta - \zeta'\right).$$

Potential well



Equivalent Sine-Gordon equation

$$\psi_1 = u e^{i(\phi_s+\phi_a)/2}\cos(\theta)$$
$$\psi_2 = u e^{i(\phi_s-\phi_a)/2}\sin(\theta),$$

- Canonical momentum: $\pi = \partial_\tau \phi_a / 4\gamma_{sa}$,
- Commutators: $\left[\phi_a(\zeta), \pi(\zeta')\right] = i\delta^D(\zeta - \zeta')$.
- Sine-Gordon equation:

$$\nabla^2 \phi_a - \partial_{\zeta_0\zeta_0}\phi_a + \tilde{\alpha}\sin\phi_a = 0$$

1D Universe: BEC time evolution

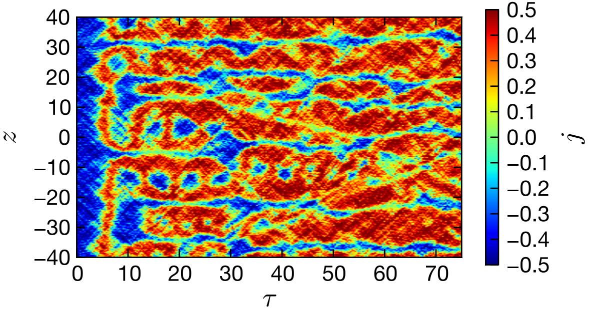


Figure : 1D space-time dynamics. Coupling $\tilde{v} = 0.1$.

2D Universe: BEC time evolution

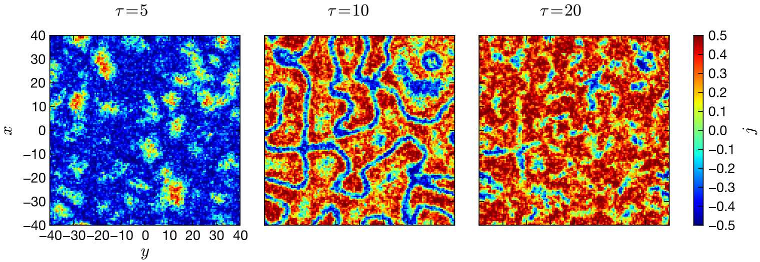


Figure : 2D density evolution. Coupling $\tilde{\nu} = 0.1$.

3D Universe: BEC simulations

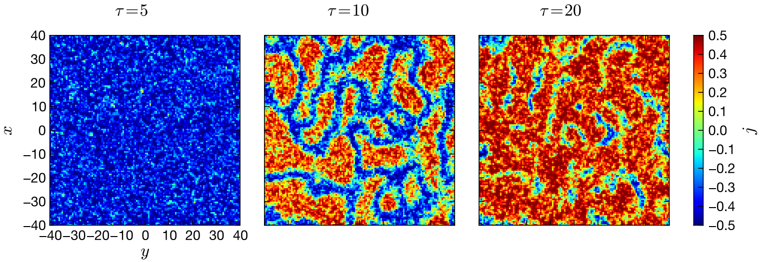


Figure : Plot of 3D density evolution. Coupling $\tilde{\nu} = 0.1$

What about relativity?

General law:

$$\partial_t^2 \phi - \frac{1}{a^2(t)} \nabla^2 \phi + dH \partial_t \phi = -\partial_\phi V(\phi),$$

where d is dimensionality, a is the scale factor and $\dot{a}/a = H$ is the Hubble constant. BEC model, including local fluctuations in the valleys:

$$\partial_t^2 \phi_a - c^2 \nabla^2 \phi_a + \frac{4\nu^2 \xi}{\hbar^2 c} \partial_t \phi_a = -\partial_{\phi_a} V(\phi_a).$$

ie, $H = \approx 3\nu^2/\hbar mc^2$.

What about metastability?

By varying the tunnel coupling ν periodically in time, it is possible to stabilize and establish the unstable vacuum at $\phi_a = \pi$ as a local minimum, which would correspond to a false vacuum in the sense of Coleman. The truncated Wigner representation then is applied to study the fate of this vacuum quantum mechanically. To stabilize the unstable vacuum we consider rapid oscillations of the tunnel coupling $\nu_t = \nu + \delta \hbar \omega \cos(\omega t)$, where frequency of oscillations $\omega \gg \omega_0 \equiv 2\sqrt{\nu g \rho_0}/\hbar$ and δ is some parameter to be fixed later. This allows us to create models that are either metastable or not, and to compare their predictions.

Conclusions

- We can create a universe in a vacuum chamber.
- Will it have tiny worlds, with people attending tiny physics symposia?
- When those tiny people observe the world around them, do they collapse our wavefunction?
- Of course this will not happen, but it is worth thinking about!
- Can we collapse the wavefunction of the universe?
- **Stay tuned for further ‘universal’ developments**

Wigner references

References:

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